M6b: Water Surface Profiles and Hydraulic Jumps

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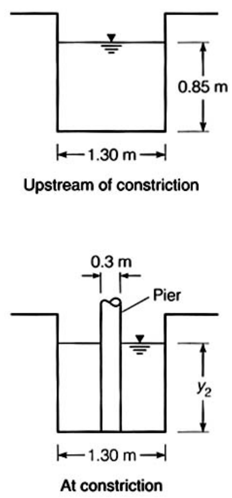
Example 3.10 (Chin 2006): Constriction in Channel



A rectangular channel 1.30 m wide carries 1.10 m^3/sec of water at a depth of 0.85 m.

a) If a 30 cm wide pier is placed in the middle of the channel, find the elevation of the water surface at the constriction.

b) What is the minimum width of the constriction that will not cause a rise in the upstream water surface?

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a) Neglecting the energy losses between the constriction and the upstream section, the energy equation requires that the specific energy at the constriction be equal to the specific energy at the upstream section:

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

Where Section 1 is the upstream section and Section 2 is the constricted section. Therefore y_1 = 0.85 m and:

$$V_1 = \frac{Q}{A_1} = \frac{1.10 m^3/\sec}{(0.85m)(1.30m)} = 1.00 m/s$$

The specific energy at Section 1 is:

$$E = y_1 + \frac{V_1^2}{2g} = 0.85m + \frac{(1.00 m/\sec)^2}{2(9.81 m/\sec^2)} = 0.901m$$

Equating the specific energies at Sections 1 and 2 yields:

$$0.901m = y_2 + \frac{Q^2}{2gA^2} = y_2 + \frac{(1.10 m^3/\sec)}{2(9.81 m/\sec^2)[(1.30m - 0.30m)y_2]^2}$$

Which simplifies to:

$$y_2 + \frac{0.0617}{y_2^2} = 0.901$$

There are three solutions to this equation:

$$y_2 = 0.33m;\ 0.80m;\ -0.23m$$

Of the two positive depths, the correct one must correspond to the same flow condition as upstream. At the upstream section:

$$Fr_1 = \frac{V_1}{\sqrt{g y_1}} = 0.35$$

The upstream flow is therefore subcritical.

$$y_2 = 0.80m \qquad Fr_2 = \frac{V_2}{\sqrt{g y_2}} = 0.49$$

Subcritical

$$if \;\; y_2 = 0.33m; \; Fr_2 = 1.9$$

Supercritical (not possible)

b) The minimum width of constriction that does not cause the upstream depth to change is associated with the critical flow conditions at the constriction. Under these conditions (and for a rectangular channel):

$$E_1 = E_2 = E_c = \frac{3}{2} y_c = \frac{3}{2}\left(\frac{q^2}{g}\right)^{1/3}$$

If b is the width of the constriction that causes critical flow, then:

$$E_1 = \frac{3}{2}\left[\frac{(Q/b)^2}{g}\right]^{1/3} \qquad 0.901m = \frac{3}{2}\left[\frac{(1.10m^3/\sec)^2}{b^2 (9.81m/\sec^2)}\right]^{1/3}$$

and *b*=0.75m. If the constricted channel is less than 0.75 m, then the flow will be choked and the upstream depth will increase.

Bridge Hydraulics

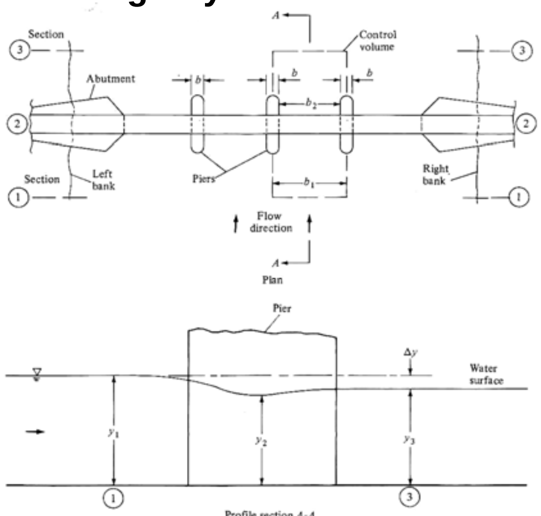

Figure 11–12 Bridge definitional sketch and water surface profiles.

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Yarnell tested different types of model piers to obtain the following equation that predicts the increase in water surface elevation (y_3) due to the bridge:

$$\frac{\Delta y}{y_3} = K Fr_3^2 \left(K + 5Fr_3^2 - 0.6\right)\left(\alpha + 15\alpha^4\right)$$

$$where \;\; \alpha = \frac{b}{b_1} \;\; the\ ratio\ of\ the\ pier\ width\ to\ the\ span\ of\ the\ piers$$

$$and \;\; \Delta y \;\; is\ the\ increase\ in\ water\ surface\ elevation\ due\ to\ the\ bridge$$

Since the flow is subcritical, the downstream depth and Froude number would be known (from computation of the backwater profile from some known location downstream). K is selected based on the pier geometry, as shown on the following table.

Table 11–2 VALUES OF *K* FOR THE YARNELL EQUATION

Pier Shape	K
Semicircular nose and tail	0.90
Lens-shaped nose and tail	0.90
Twin-cylinder piers with connecting diaphragm	0.95
Twin-cylinder piers without diaphragm	1.05
90 deg triangular nose and tail	1.05
Square nose and tail	1.25

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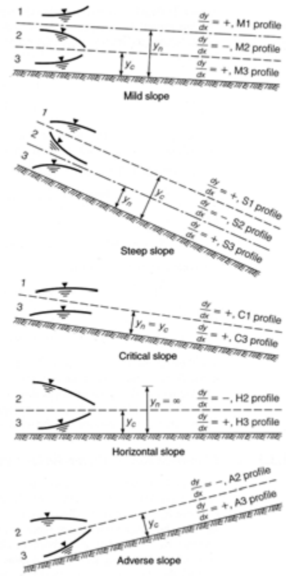

Water Surface Slope Classifications (Figure 3.11, Chin 2006)

Hydraulic Classification of Slopes (Table 3.5, Chin 2006)

Name	Type	Condition
Mild	M	$y_n > y_c$
Steep	S	$y_n < y_c$
Critical	C	$y_n = y_c$
Horizontal	H	$y_n = \infty$
Adverse	A	$S_o < 0$

Mild slope because $y_n > y_c$

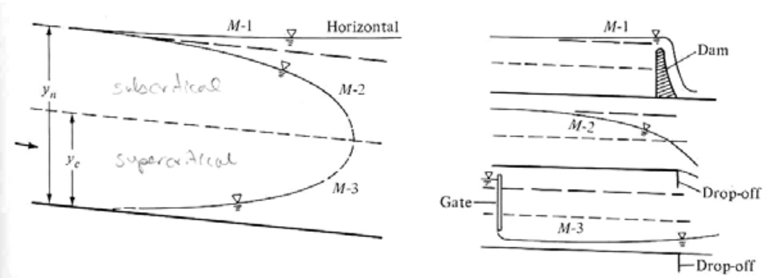

Figure 7–18 Gradually varied flow on a mild slope.

M – 1: above both y_n and y_c (above both)
M – 2: between y_n and y_c (between the upper two)
M – 3: below both y_n and y_c (below both)

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Steep slope because $y_c > y_n$

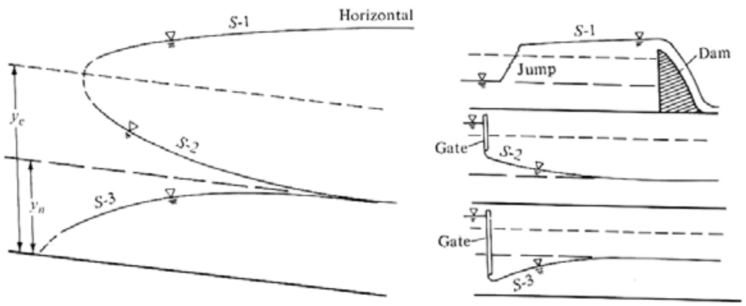

Figure 7–19 Gradually varied flow on a steep slope.

S – 1: above both y_n and y_c (above both)
S – 2: between y_n and y_c (between the upper two)
S – 3: below both y_n and y_c (below both)

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Critical slope because $y_c = y_n$

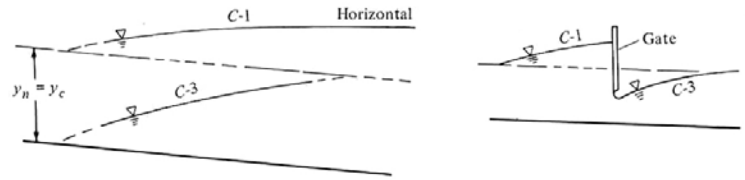

Figure 7–20 Gradually varied flow on a critical slope.

C – 1: above both y_n and y_c (above both)
There is no C – 2: not possible to be between y_n and y_c (they are the same)
C – 3: below both y_n and y_c (below both)

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Horizontal slope because y_n is infinite

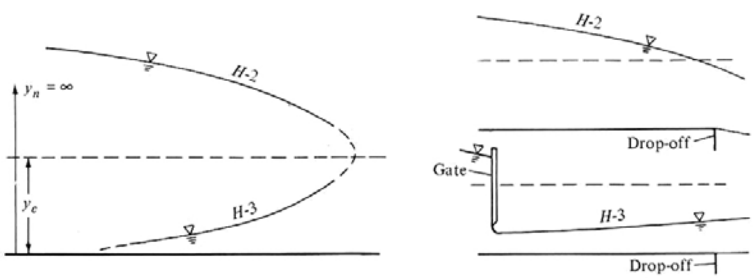

Figure 7–21 Gradually varied flow on a horizontal slope.

There is no H – 1: not possible to be above both (y_n at infinity)
H – 2: between y_n and y_c (between both)
H – 3: below both y_n and y_c (below both)

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Adverse slope because y_n is infinite and channel slopes upwards

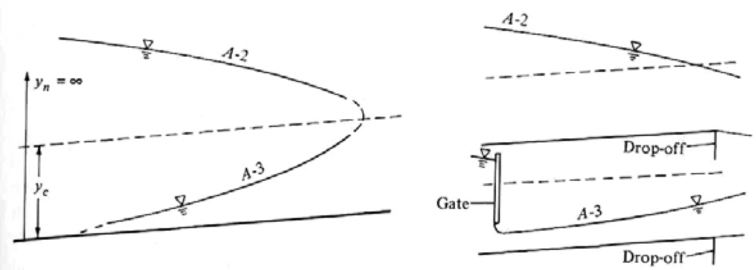

Figure 7–22 Gradually varied flow on an adverse slope.

There is no A – 1: not possible to be above both (y_n at infinity)
A – 2: between y_n and y_c (between both)
A – 3: below both y_n and y_c (below both)

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TABLE 7–2 SUMMARY OF WATER SURFACE PROFILES

Channel	Depth	Froude number	Slope	dy/dx	Control upstream/ downstream	Curve type
Mild	$y > y_n > y_c$	Fr < 1	$S_f < S_0$	+	DS	M–1
Mild	$y_n > y > y_c$	Fr < 1	$S_f > S_0$	−	DS	M–2
Mild	$y_n > y_c > y$	Fr > 1	$S_f > S_0$	+	US	M–3
Steep	$y > y_c > y_n$	Fr < 1	$S_f < S_0$	+	DS	S–1
Steep	$y_c > y > y_n$	Fr > 1	$S_f < S_0$	−	US	S–2
Steep	$y_c > y_n > y$	Fr > 1	$S_f > S_0$	+	US	S–3
Critical	$y > y_n = y_c$	Fr < 1	$S_f < S_0$	+	DS	C–1
Critical	$y_n = y_c > y$	Fr > 1	$S_f > S_0$	+	US	C–3
Horizontal	$y > y_c$	Fr < 1	$S_f > S_0 = 0$	−	DS	H–2
Horizontal	$y_c > y$	Fr > 1	$S_f > S_0 = 0$	+	US	H–3
Adverse	$y > y_c$	Fr < 1	$S_f > S_0$	−	DS	A–2
Adverse	$y_c > y$	Fr > 1	$S_f > S_0$	+	US	A–3

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Computation of Water Surface Profiles

The equation describing the shape of the water surface profile in an open channel is derived from the energy equation and written in the form:

$$S_o - S_f = \frac{\Delta\left(y + \alpha \dfrac{V^2}{2g}\right)}{\Delta x}$$

Rearranging leads to:

$$\Delta L = \frac{\left[y + \alpha \dfrac{V^2}{2g}\right]_2^1}{\bar{S}_f - S_o}$$

which describes the distance between upstream and downstream sections.

Example 7-11 (Prasuhn 1987) Direct Step Method to Calculate Water Surface Profiles

A discharge of 800 cfs occurs in a long rectangular channel that is 20 ft wide and has a slope of 0.0005 (= $5\text{x}10^{-4}$). The channel ends in an abrupt drop-off. The Manning's n is 0.018. Calculate the water surface profile from the drop-off to a distance at which the depth has reached 99 percent of the normal depth.

The normal and critical depths are needed to determine the type of curve. From the Manning equation:

$$Q = 800\,ft^3/\sec = \frac{1.49 A R^{2/3} S_o^{1/2}}{n} = \frac{1.49\left[(20\,ft)y_n\right]^{5/3}(0.0005)^{1/2}}{(0.018)(20\,ft + 2y_2)^{2/3}}$$

and the normal depth at y_2 is determined to be $y_n = 8.01$ ft

The unit width discharge and the critical depth values are:

$$q = \frac{800\,ft^3/\sec}{20\,ft} = 40 cfs/\,ft$$

$$y_c = \sqrt[3]{\frac{(40 cfs/\,ft)^2}{32.2\,ft/\sec^2}} = 3.68\,ft$$

The water surface profile will therefore be a M-2 curve. The depth at the brink (the drop off location) will be:

$$y_{brink} \approx 0.7 y_c = 2.58\,ft$$

and the distance from the brink to the critical depth will be:

$$\Delta x_1 \approx 4 y_c = 4(3.68\,ft) = 15\,ft$$

Example 7-10 (Prasuhn 1987)

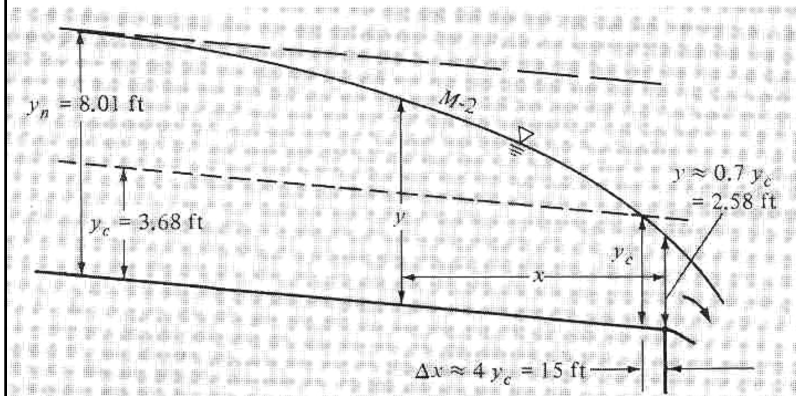

Figure 7–23 M-2 curve for Example 7–10.

The distances between the downstream and upstream sections are calculated to be:

$$\Delta x = \frac{\left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right)}{S_f - S_o}$$

And the friction slopes for each segment can be calculated using the Manning's equation, using average n, V, and R values for each reach:

$$S_f = \left(\frac{n_{av} V_{av}}{1.49 R_{av}^{2/3}}\right)^2$$

The computations are for a distance upstream of the drop off until the depth is 0.99 (8.01 ft) = 7.93 ft.

The computations are carried out in the following table:

y (ft)	A (ft2)	R (ft)	V (ft/s)	$y+V^2/2g$	$\Delta(y+V^2/2g)$	R_{av} (ft)	V_{av} (ft/s)	S_f	Δx (ft)	$X=\Sigma\Delta x$ (ft)
3.68	73.6	2.68	10.87	5.515	--	--	--	--	--	15
4.68	93.6	3.19	8.55	5.815	0.300	2.94	9.71	$3.27\text{x}10^{-3}$	108	123
5.68	113.6	3.62	7.04	6.450	0.635	3.41	7.79	$1.72\text{x}10^{-3}$	518	641
6.68	133.6	4.00	5.99	7.237	0.787	3.81	6.52	$1.04\text{x}10^{-3}$	1449	2090
7.68	153.6	4.34	5.21	8.101	0.864	4.17	5.62	$6.82\text{x}10^{-4}$	4750	6840
7.93	158.6	4.42	5.04	8.324	0.233	4.38	5.13	$5.36\text{x}10^{-4}$	6212	13,052

Hydraulic Jumps

A hydraulic jump is a steady, nonuniform phenomenon that occurs in open channels when a supercritical flow encounters a deeper subcritical flow. As the supercritical flow encounters the slower moving water, it tends to flow under it and then spread upward, creating a large eddy or roller. This can dissipate a significant amount of energy and is desirable below a spillway or sluice gate before the flow enters a natural channel, reducing erosion potential.

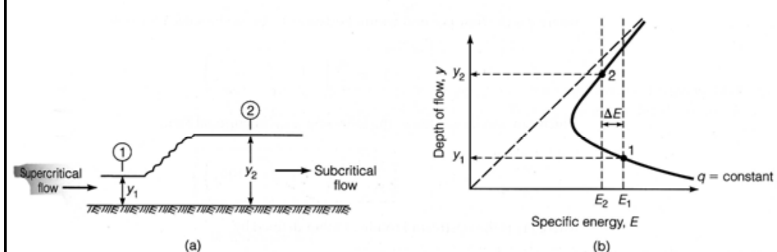

Figure 3.12, Chin 2006

Characteristics of Hydraulic Jumps (Table 4.5, Chin 2000)

Fr_1	Jump characteristics
1–1.7	standing wave (*undular jump*); kinetic-energy loss (as a percentage of the upstream kinetic energy, $V_1^2/2g$) < 5%
1.7–2.5	smooth surface rise (*weak jump*); kinetic-energy loss 5%–15%
2.5–4.5	unstable oscillating jump, where each irregular pulsation creates a large wave that can travel far downstream, damaging earth banks and other structures (*oscillating jump*); kinetic-energy loss 15%–45%
4.5–9.0	stable jump, best performance and action, and insensitive to downstream conditions (*steady jump*); kinetic-energy loss 45%–70%
> 9.0	rough, somewhat intermittent (*strong jump*); kinetic-energy loss 70%–85%

Source: Gerhart et al. (1992).

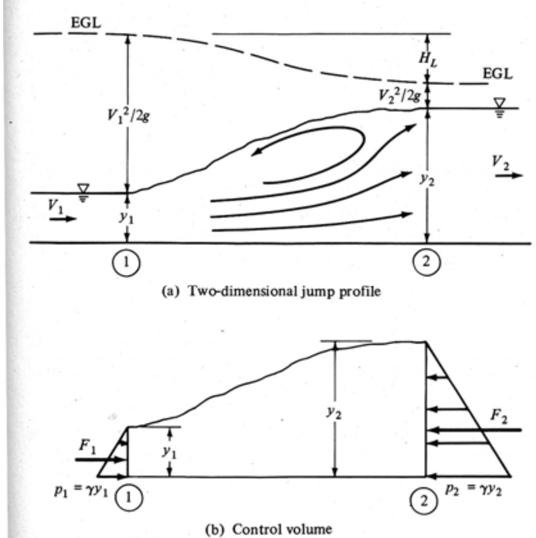

Figure 7–13 Hydraulic jump.

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The forces in the flow direction consist of the hydrostatic forces:

$$\frac{\gamma y_1^2 b}{2} - \frac{\gamma y_2^2}{2} = \rho Q(V_2 - V_1)$$

This can be used to derive the following that can be used to predict the conjugate (required) downstream depth of a hydraulic jump:

$$\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1+8Fr_1^2} - 1\right)$$

The following graph can also be used to predict the depths, the length of the jump, and the head loss in the jump.

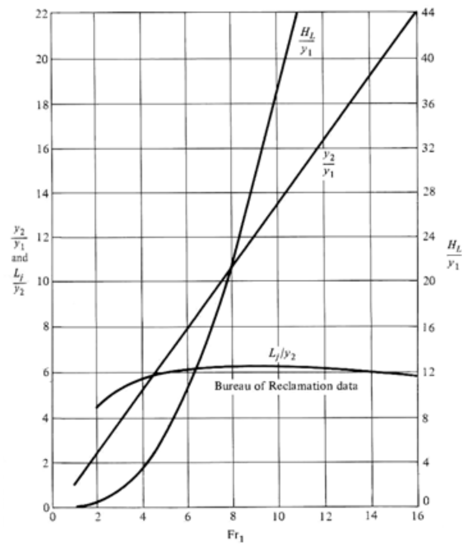

Figure 7–14 Characteristics of the hydraulic jump.

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Example 7-7 (Prasuhn 1987) Hydraulic Jump Characteristics

What is the downstream depth required for a jump to occur in a 20 ft wide rectangular channel, if the upstream depth is 2 ft and the discharge is (a) 200 cfs, or (b) 640 cfs. What is the head loss in each case?

For 200 cfs:

$$V_1 = \frac{Q}{A} = \frac{200\, ft^3/\sec}{(2\, ft)(20\, ft)} = 5\, ft/\sec$$

$$Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{5\, ft/\sec}{\sqrt{(32.2\, ft/\sec^2)(2\, ft)}} = 0.62$$

The upstream flow is subcritical and a hydraulic jump will not occur.

For 640 cfs:

$$V_1 = \frac{Q}{A} = \frac{640\, ft^3/\sec}{(2\, ft)(20\, ft)} = 16\, ft/\sec$$

$$Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{16\, ft/\sec}{\sqrt{(32.2\, ft/\sec^2)(2\, ft)}} = 1.99$$

The upstream flow is therefore supercritical, and a jump will occur. The resulting downstream depth can be calculated:

$$\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1+8Fr_1^2} - 1\right) = \frac{y_2}{2\, ft} = \frac{1}{2}\left(\sqrt{1+8(1.99)^2} - 1\right)$$

$$y_2 = 4.72\, ft$$

The resulting energy loss is therefore:

$$H_L = H_1 - H_2 = \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right)$$

$$H_L = \frac{(y_2 - y_1)^3}{4y_1y_2} = \frac{(4.72\, ft - 2\, ft)^3}{4(2\, ft)(4.72\, ft)} = 0.53\, ft$$

Combined Water Surface Profiles

Water surface profiles are the result of an obstruction to the flow or some change or changes to the channel itself, such as its slope. The transitions that may result involve distinct profiles as illustrated in the following profiles:

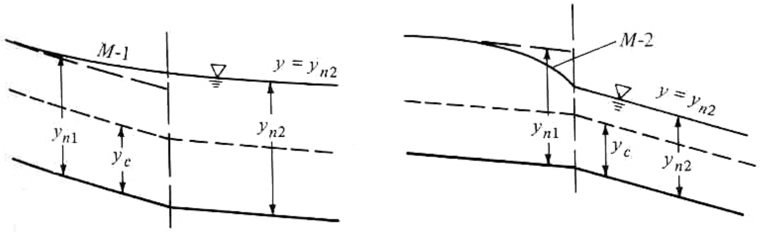

Figure 7–24 Transitions from one mild slope to another.

Always subcritical; the required downstream control is provided by the downstream normal water depth, a friction control because it is related to the channel friction through the normal depth and the Manning equation.

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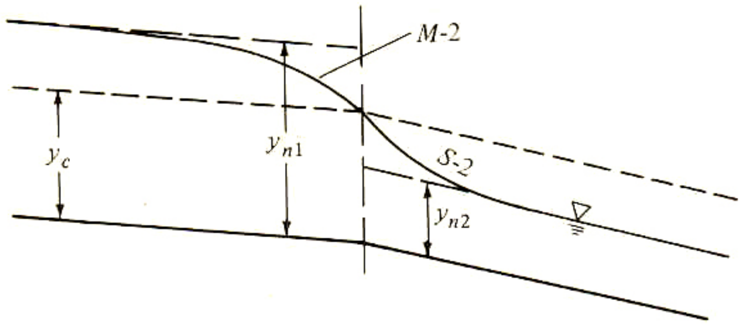

Figure 7–25 Transition from mild to steep slope.

The upstream subcritical flow must have a downstream control, while the downstream supercritical flow must have an upstream control. Therefore, the transition curve must pass through the critical depth at the grade break (the only possible control for both directions).

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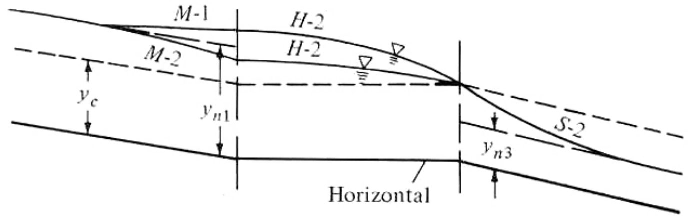

Figure 7–26 Transition from mild to horizontal to steep slope.

This transition is similar to the above example: the control will be the grade change following the horizontal section (the critical depth). Depending on the length of the horizontal section, the H-2 curve can either be above or below the normal depth of the mild slope.

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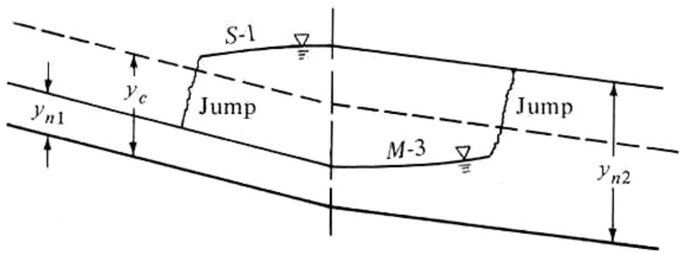

Figure 7–27 Transition from steep to mild slope.

The transition from a steep to a mild slope cannot be accomplished by the water profiles alone and requires a hydraulic jump. The equation describing the ratio of y_2/y_1 can be applied at the break in the grade with y_1 taken as the upstream normal depth. The y_2 conjugate depth can be compared to the downstream normal depth. If y_2 is greater than the downstream normal depth, the jump must be downstream of the break, otherwise it is upstream of the break. This is because y_2 decreases as y_1 increases for constant channel and discharge conditions.

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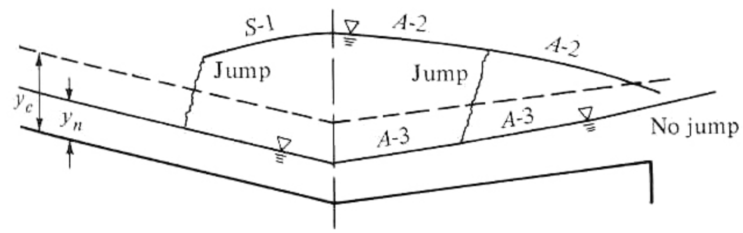

Figure 7–28 Transition from steep to adverse slope.

This is a more complicated example. A jump may occur in either section, but since the adverse section ends with a drop off, there is the possibility that no jump will occur.

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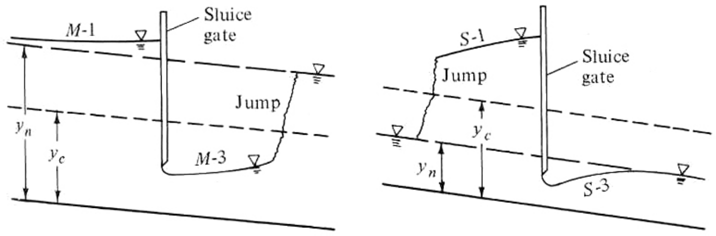

Figure 7–29 Profiles caused by a sluice gate.

On a mild slope, the flow will be at the normal depth until the gate is lowered into the flow. Immediately thereafter, it will act as a control on the upstream section. As the gate is lowered below the critical depth, it will act as a control on the downstream section also, and a hydraulic jump will occur. If the gate remains above the critical depth, there will be little energy loss and the downstream depth will remain at nearly normal depth.

On a steep slope, the instant the gate enters the flow, a surge will form that moves upstream some distance and become stationary. The gate acts as both a control for the upstream subcritical flow and the downstream supercritical flow.

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